Water limited crop growth and LINTUL2

LINTUL2: a simple general crop growth model for water-limited growing conditions

(example: spring wheat)

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This is a reworked text based on work from Marcel van Oijen and Peter Leffelaar. The implementation of the LINTUL models in FST or R can be freely downloaded from models.pps.wur.nl.

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INTRODUCTION

LINTUL2 describes production (as applied to spring wheat) under water-limited conditions by including a water balance in the LINTUL1 model. Conditions are still optimal with respect to other growth factors, i.e. ample nutrients and a pest-, disease- and weed-free environment. The water balance determines how much soil water is available for the crop, whereas potential transpiration rates and soil characteristics determine how a limited soil water supply affects crop growth rates. Further, drought also affects crop emergence, LAI growth in early growth stages and increments in rooting depth.

With the LINTUL2 model, options for water conservation can be studied, as well as differences among cultivars in drought tolerance. LINTUL2 can only be understood on the basis of LINTUL1, the crop growth model for potential production. The effect of water deficiency on crop growth is transmitted through two variables: (1) the transpiration reduction factor (TRANRF) acting on total crop growth; and (2) the root modification factor (rMOD) acting on root-shoot partitioning.

To understand how this works, we first describe the physical processes related to evapotranspiration rates that determines the demand for water, i.e. the amount of water needed per day for crop growth rates without water deficiency. LINTUL 2 uses the Penman equation to estimate the potential evapotranspiration in combination with (a simple version of) the tipping bucket model to account for the amount of water available in the soil for crop uptake and soil evaporation. Hence, we will only cover the Penman equation and the tipping bucket as used in LINTUL2 extensively, although other and more detailed approaches exists.

This text contains many equations that provide insight in the processes and the steps in the calculation procedure. <u>These you do not need to learn by heart</u>! Focus on understanding how the environmental factors determine potential production and how water availability can affect evaporation and transpiration rates. It is important to understand how drought interacts with crop growth, i.e. how it can reduce crop growth rates, which affect growth of particular organs depending on crop development. Hence. drought may affect LAI and light interception and therefore crop growth rates in later parts of the season as well. The general principles are explained in the main text. In the boxes, additional details are provided that are not examined.

PHYSICAL PROCESSES RELATED TO EVAPOTRANSPIRATION

<u>Potential evapotranspiration</u>[1]

Penman (1948) was the first to describe evapotranspiration (ET) in physical-mathematical terms. He derived equations that describe evaporation (E) and transpiration (T) from free-water surfaces, bare soil and short grass swards for 10-day periods. His equations were based on a so-called lysimeters, columns filled with soil where drainage, water content and evapotranspiration amounts per unit area could be measured on a daily basis. He used a simplified heat balance to solve the mathematical equations. A heat balance includes terms for sensible heat, latent heat, stored heat and heat exchange. Penman ignored the changes in stored heat

[1] This text is partially based on Goudriaan and van Laar, 1994, Modelling potential crop growth processes, and on Lövenstein *et al.*, 1995, Principles of Production Ecology.

and the horizontal heat exchange between the column and the surrounding soil to solve the equations. For longer periods, such as a growing season, these assumptions are reasonable while for shorter periods of a few days they probably are not. There is a continuous heat exchange between the lysimeter, the surrounding soil and the air. For longer time periods, soil and air temperatures are in balance with a small temperature gradient and consequently a small net heat exchange. Soils, and especially wet soils, may store large amounts of heat. Hence, soil temperatures respond slowly to changes in air temperatures, especially at depth, and a sudden change in air temperature will result in a strong temperature gradient and a large net heat exchange that may either increase or decrease the energy available for evapotranspiration.

Potential evapotranspiration is often calculated by a Penman-type equation (Penman, 1948), but also other methods are used, depending on the objective, situation and data availability (Kraalingen and Stol, 1997). Nowadays, the Penman-Monteith equation (Allen *et al.*, 1998) is often used which includes crop specific resistances that may improve the drying power term. Here, only Penman is discussed.

Penman defined potential evapotranspiration in 1956 as the evapotranspiration[2] from "*a fresh green crop, of about the same colour as grass, completely shading the ground, of fairly uniform height, and never short of water*" (cf. Howell and Evett (2004)), and derived Eq. (1):

$$ET = \frac{\Delta}{\Delta+\gamma}\frac{R_{net}}{\lambda} + \frac{\gamma}{\Delta+\gamma}\frac{E_{air}}{\lambda} \qquad (1)$$

where the potential evapotranspiration of water, ET (kg H_2O m^{-2} d^{-1} or mm d^{-1}, see footnote[3]) from both crop and soil, is the result of a weighted sum of a ***net radiation term*** (R_{net}, MJ m^{-2} d^{-1}) and an aerodynamic term (E_{air}) or ***drying power term*** (both terms are explained in more detail below). Both terms are converted into water evaporation by dividing them by the heat of vaporization (λ) expressed in MJ kg^{-1} H_2O (the value of which is 2.45 around 20˚C). The adiabatic psychrometer coefficient (γ) is a measure of the increase in water vapour pressure in air in exchange for a 1 ˚C decrease of air temperature. Its value is 0.67 hPa ˚C^{-1} at about 20 ˚C (footnote[4]). The term adiabatic indicates that it reflects a process where energy is maintained in the system, *i.e.* heat is converted from air to water vapour without energy losses. The derivative or slope of the saturated vapour pressure curve, Δ, has the same units as this psychrometer coefficient.

The empirically determined saturated vapour pressure (e_s) curve is shown in Figure 1. The solid line reflects the fitted equation e_s (eq. 2):

$$e_s = 6.11 \times e^{\left\{\frac{(17.47 \times T_S)}{(239+T_S)}\right\}} \qquad (2)$$

[2] Penman spoke of potential evaporation, but today the term evapotranspiration is commonly used.

[3]). One cubic meter of water contains 1000 liter of H_2O. This one cubic meter can be seen as one square meter of surface area covered with a water layer of 1 m, or 1000 mm. Hence, 1 liter m^{-2} equates to a water layer of 1 mm.

[4] Approximately 2450 J is needed to vaporize 1 g of water (λ), while the volumetric heat capacity of air is 1200 J m^{-3} ˚C^{-1}. This means that about 0.5 g of water can be added to the air as vapour in exchange for a temperature decrease of 1 ˚C of that same air. According to the gas law, 0.5 g water per m^3 of air corresponds to a pressure of 67 Pa at a temperature of 20 ˚C ($P = c\,R\,T\,/\,M_{H2O}$, with P: Pa; c: g m^{-3}; R: 8.3142 Pa m^3 mol^{-1} K^{-1} (universal gas constant); T temperature in K; M_{H2O} molecular weight of water, g mol^{-1}).

The derivative of e_s in eq. (2), $\frac{de_s}{dT}$, is represented by the Greek letter delta (Δ) and has the unit hPa $^oC^{-1}$ (eq. 3):

$$\Delta = \frac{4175.3 \times \left(6.11 \times e^{\left\{\frac{(17.47 \times T_S)}{(239 + T_S)}\right\}}\right)}{(239+T_S)^2} = e_S \frac{4175.3}{(239+T_S)^2} \tag{3}$$

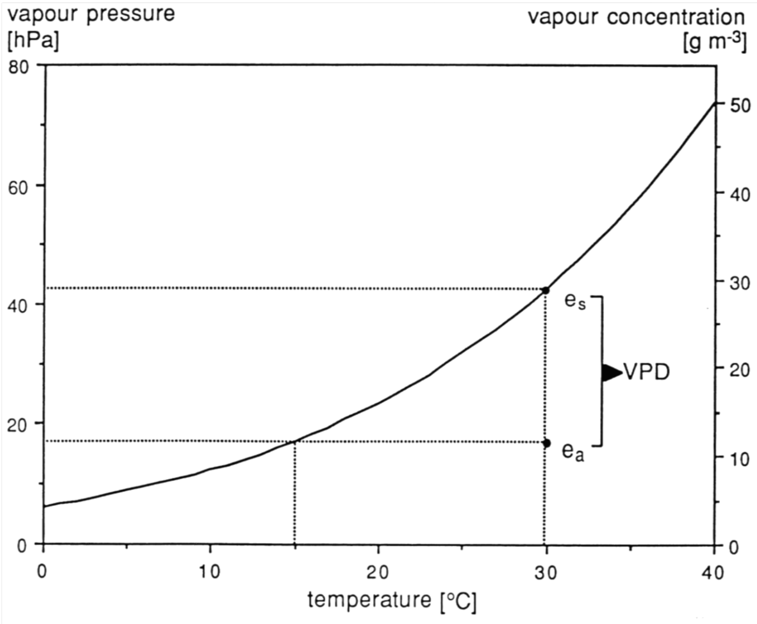


Figure 1. Empirical relationship between temperature, T (T_s: temperature for water vapour saturated air), and saturated vapour pressure, e_s, (left y-axis, hPa from equation 2) and saturated vapour concentration (right y-axis, g m^{-3}). The ratio between the actual vapour pressure in air, e_a, and the value at saturation, e_s, at a certain temperature, reflects the relative air humidity, RH. The difference between e_s and e_a is the vapour pressure deficit (VPD).

Example: at 15˚C: $RH = e_a/e_s$=1, VPD = 0; at 30˚C: $RH = e_a/e_s$=18/42=0.43, VPD = 24 hPa.

The Penman equation (1) shows that to evaporate water, energy is needed and produced water vapour needs to be removed by either diffusion of wind, reflected by E_{air}, the drying power term. A transpiring leaf is typically cooler than the surrounding air, indicating that the energy needed is derived from radiation, the term with R_{net}, and from some cooling of the surrounding air. Note that energy from incoming radiation that is not used for evapotranspiration (latent heat) will warm up the air (sensible heat). So when the air is cold Δ will be smaller when compared to a warmer air. This means that the proportion of energy for evapotranspiration increases with temperature and a larger proportion of the radiation will be used to warm the air when it is cold.

Radiation power term

The energy to evaporate water is supplied by incident global short wave radiation, R_s. About 98% of the radiation emitted by the sun is in the waveband from 300-3000 nm (short-wave radiation). All incident radiation, composed of ultra-violet radiation (**UV**; 300-400 nm), photosynthetically active radiation (**PAR**; 400-700 nm) and near-infrared radiation (**NIR**; 700-3000 nm), is an energy source for evaporation. For crop growth modelling purposes, it is generally assumed that both PAR and NIR have equal shares of about 50% in the total spectrum, thus ignoring the small amount of UV (about 4%). In the near-infrared region of the spectrum most of the radiation is scattered by leaves. Reflection and transmission share their portion rather equally, Table 1.

Table 1. Percentage of absorbed, reflected and transmitted radiation by a leaf for photosynthetically active radiation (PAR) and near infrared radiation (NIR).

Spectral range	Absorbed	Reflected	Transmitted
PAR	80	10	10
NIR	20	40	40
50% PAR + 50% NIR	50	25	25

The reflection of NIR reduces the heat load from wavelengths that are not used for CO_2-assimilation. As a result, only about 50% of the incident global radiation is absorbed by a single green leaf compared to 80% for PAR (Table 1). A part of the incoming global radiation is reflected by soil and canopy, usually denoted as **albedo** (α) in the literature. Table 2 lists some typical albedo values for different surfaces.

Table 2. Approximate albedo values (in %) for different surfaces.

Grass (a cut lawn)	Forest	Soil	Desert soil	Water body
25	10	10	30	5

In the **far-infrared** region of the spectrum (between 3000 and 30.000 nm) surfaces behave like black bodies that not only absorb all incident long-wave radiation, but also emit radiation. This emitted (so-called) thermal**)** **long-wave radiation** is higher than the amount received as radiation from the sun which means that less energy is left for evapotranspiration, Eq. (4):

$$R_{net} = (1 - \alpha)\, R_s - R_{netl} \tag{4}$$

where R_{netl} is the **net outgoing long wave radiation** in MJ m^{-2} d^{-1}. Any surface above the absolute minimum temperature (–273.15 ˚C or 0 K) emits thermal radiation, proportional to the fourth power of the absolute temperature according to the law of Stefan-Boltzmann, Eq. (5):

$$R_{netl} = \sigma \times (273.15 + T)^4 \tag{5}$$

where σ is the **Stefan-Boltzmann constant** 5.668 10^{-8} W m^{-2} K^{-4} or 4.897 10^{-9} MJ d^{-1} m^{-2} K^{-4}. Since the earth with its vegetation at terrestrial temperatures is warmer than the sky outside the troposphere, there is a net upward flux of long-wave radiation. This flux is larger under clear than under overcast skies, as clouds with temperatures approaching those at earth, are (downward) radiating surfaces too (Chang, 1968, p.166).

Commonly available data from weather stations include measurements of global radiation but do often not have data about the number of sunshine hours. Therefore, this **effect of clouds on the flux** is corrected for using the relative air humidity by the Brunt (1932) formula:

$$flux\ correction = max\left(0, 0.55 \times \left(1 - \frac{e_a}{e_s}\right)\right) \tag{6}$$

Combining equation 4, 5 and 6 gives the expression for the **net radiation**:

$$R_{net} = (1-\alpha) \times R_S - \sigma \times (273.15 + T)^4 \times max\left(0, 0.55 \times \left(1 - \frac{e_a}{e_s}\right)\right) \quad (7)$$

The **albedo** α from soil and crop canopy are about 0.15 and 0.25 respectively and, consequently, R_{net} also differs:

$$R_{net\_soil} = (1-0.15) \times R_S - \sigma \times (273.15 + T)^4 \times max\left(0, 0.55 \times \left(1 - \frac{e_a}{e_s}\right)\right) \quad (8)$$

$$R_{net\_crop} = (1-0.25) \times R_S - \sigma \times (273.15 + T)^4 \times max\left(0, 0.55 \times \left(1 - \frac{e_a}{e_s}\right)\right) \quad (9)$$

Drying power term

Transpiration of water from a leaf and **evaporation** of water from a soil surface are diffusion processes that can, in principle, be described by: diffusion flux = (gradient in vapor concentration) / (resistance), or formally, for leaves:

$$E_{air-leaves} = \frac{[H_2O]_{int} - [H_2O]_{ext}}{r_s + r_b} \quad (10)$$

where $E_{air-leaves}$ is the actual transpiration rate in g H_2O m^{-2} leaf s^{-1}; $[H_2O]_{int}$ vapour concentration inside the stomatal cavity in g H_2O m^{-3} air; $[H_2O]_{ext}$ vapour concentration in the open air in g H_2O m^{-3} air; r_s stomatal resistance to water vapour in s m^{-1}; and r_b the boundary layer resistance to water vapour in s m^{-1}, Figure 2.

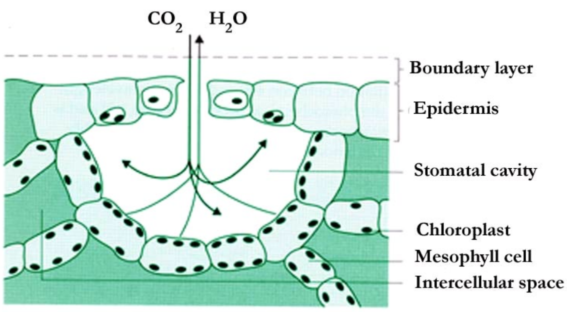


Figure 2. A portion of a leaf showing a stomatal cavity, the exchange processes of CO_2 and H_2O and the boundary layer above the leaf surface.

Equation 10 shows that the transpiration rate is proportional to the vapour concentration difference between the stomatal cavity and the open air, ($[H_2O]_{int}$ – $[H_2O]_{ext}$), and a conductivity, which is the inverse of the sum of the resistances (r_s + r_b). Since the air inside the stomatal cavity is practically saturated with water vapour (relative humidity near 100%), it is the drying power of the air, through its lower water vapour concentration, that determines the gradient. The vapour pressure difference (e_s – e_a) is a measure for this concentration gradient in the Penman equation, whereas the boundary layer resistance is determined by an empirical wind function: a stronger wind will diminish the boundary layer and the resistance r_b will decrease (or: the conductivity will increase). The stomatal resistance is actively regulated, whereas the boundary layer thickness depends on the mixing of air around the leaves. Therefore, wind speed affects the boundary layer thickness and will also remove the water vapour that has evaporated, thus affecting the gradient.

The wind function estimates the conductance for transfer of latent and sensible[5] heat from the surface to the standard height and depends on roughness of the surface and atmospheric stability. The wind function, is defined as f(u) = 0.643 (1 + 0.54 wn) with wind speed wn in m s^{-1} measured at a standard height of 2 meter. These values originate from Penman[6] and applies to short, closed grass crops (Penman, 1956). The unit of f(u) is MJ m^{-2} d^{-1} hPa^{-1}.

The aerodynamic component in Eq. (1), E_{air}, may now be defined as

$$E_{air} = (e_s - e_a) \times 0.643 \times (1 + 0.54 \times wn) \quad (11)$$

The drying power term is applied to both soil and canopy, because both surfaces may be considered as having a relative humidity of one (RH = 1) in their pores: stomata under normal conditions contain water with a saturated vapour pressure; soil pores at a suction of even pF = 4 have still a relative humidity of 99.3 % (Koorevaar *et al.*, 1983). Clearly, possible effects of mulching or soil crust formation on evaporation are not considered here.

Attributing evapotranspiration to transpiration and evaporation

The drying and radiation power terms of the Penman equation are expressed per unit ground area, and do not take account of the size of the canopy. We thus still need to quantify the weighing factors that partition total evaporative demand between soil and crop. The proportion of the incoming radiation that is intercepted by the crop determines how much water can transpire from leaves and how much energy is left for evaporation from the soil surface. When determining how much of this incident global radiation will reach the ground surface, a different extinction coefficient (k) needs to be used when compared to the k value for PAR. NIR light will more easily transmit through leaves when compared to PAR and will also have more multiple reflections in the canopy. Hence, NIR light will more easily penetrate into the canopy and the extinction coefficient k for total global radiation is with a value of about 0.5 lower when compared to the k of 0.7 for PAR alone. With this estimate of light extinction and LAI, soil evaporation and crop transpiration can be approximated as:

$$E_{\mathrm{soil}} = e^{(-0.5 \times L)} \times ET \quad (12)$$

$$T_{\mathrm{crop}} = \left(1 - e^{(-0.5L)}\right) \times ET \quad (13)$$

[5] The term *latent* heat loss is reserved for the hidden loss of energy resulting from transpiration (or evaporation) of water from a surface. The term *sensible* heat loss refers to the warming up of leaves and air by absorbing radiation.

[6] In equation 1, the heat of vaporization, λ is shown as separate variable. However, λ is sometimes incorporated in the wind function f(u). The wind function then appears with a value of 0.263 when f(u) has the unit MJ m^{-2} d^{-1} hPa^{-1} or with a value of 2.63 when f(u) has the unit MJ m^{-2} d^{-1} kPa^{-1}. Dividing 0.643 by λ, yields 0.263, which needs to be multiplied by a factor 10 when e_s and e_a are expressed in kPa.

As shown above, the values for Rnet differ for soil and crop canopies and hence the value for ET in equations 12 and 13 should slightly differ. Further, when leaves are wet, transpiration of water from the stomata is delayed. Therefore, potential transpiration rate is reduced by half the amount of intercepted rain (as the average of values 0.3 - 1.0 reported by Singh and Szeicz (1979)). The correct equations as used in LINTUL2 are:

$$E_{\text{soil}} = e^{(-0.5\times L)} \times \left(\frac{\Delta}{\Delta+\gamma} \times \frac{R_{net\_soil}}{\lambda} + \frac{\gamma}{\Delta+\gamma} \times \frac{E_{air}}{\lambda}\right) \quad (14)$$

$$T_{\text{crop}} = max\left(0, \left(1 - e^{(-0.5\times L)}\right) \times \left(\frac{\Delta}{\Delta+\gamma} \times \frac{R_{net\_canopy}}{\lambda} + \frac{\gamma}{\Delta+\gamma} \times \frac{E_{air}}{\lambda}\right) - 0.5 \times R_{int}\right) \quad (15)$$

Water in the soil

To estimate how much water a crop can transpire, we need to know how much water the soil can supply and how easy it is for crops to take that up. Water can be found in soil pores, which range in size. In a soil saturated with water, all pores are filled with water. When a soil becomes dryer, big pores will empty first and then small ones: small pores hold on to water much stronger than big pores. The biggest pores will already empty quickly, even by gravity alone. After a short period, only medium and small pores remain filled with water when there are no plant roots. This is what we refer to as the **field capacity** of soils, expressed as the **volumetric water content** (m^3 water m^3 soil) of the soil. Plant roots can take up remaining water, except for water in very small pores which requires more suction than plant roots can apply to empty them. Small pores not only take a lot of suction to empty, but have a limited capacity to transport water. Hence extraction of water from an already rather dry soil is far more difficult than from wet soil.

The distribution of pore sizes depends on soil texture (and soil compaction, but that varies with crop management and is ignored here for simplicity). Clay soils will have a larger proportion of small pores when compared to a coarse textured soil, such as sandy soils. The actual amount of water that the particular soil contains depends therefore on the soil texture and the matric suction that is applied by gravity or plant roots as shown in Figure 3.

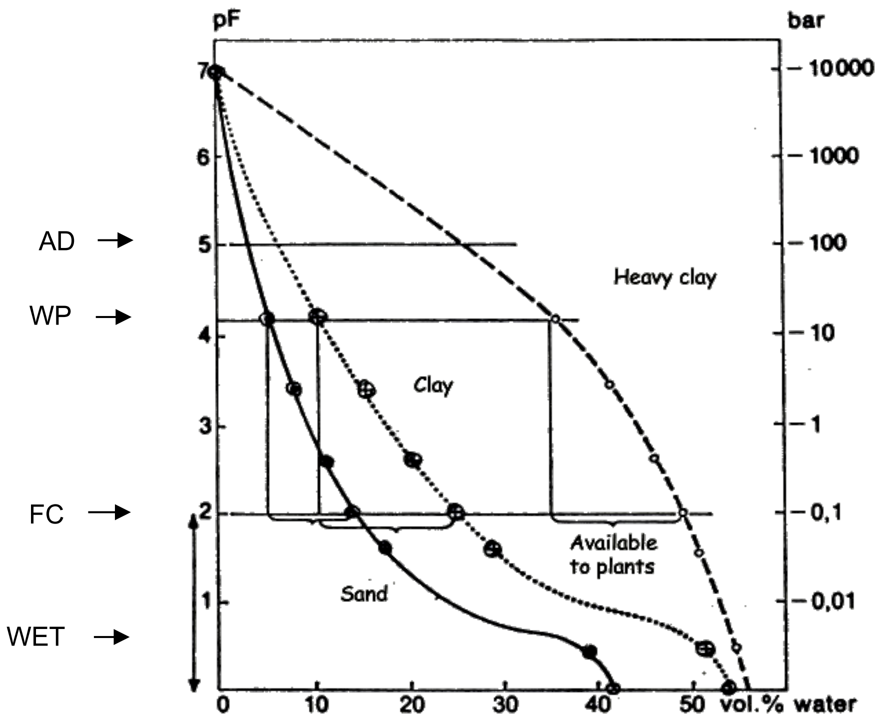
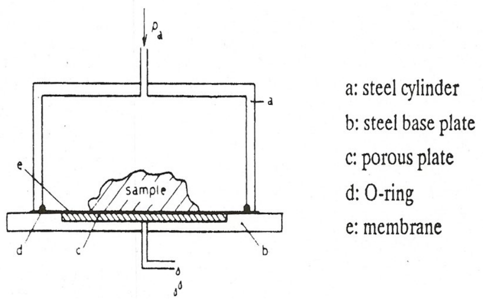


Figure 3. Soil water characteristics for sand, clay and a heavy clay soil and the indication of some important data points used in the tipping bucket model. (pF = ^{10}log{–(suction in cm hanging water column)/cm}, left y-axis; a bar = 10 m of water pressure = 1000 cm), right y-axis.)

The matric suction is expressed as **pF**, which is calculated as log(-h), with h the **hydraulic head** in cm. The hydraulic head is equivalent to the pressure that is applied on a surface by column of water in cm and can directly be interpreted as a negative pressure in hPa.

This pF value can be understood as follows. Suppose that a long u-shaped tube filled with water is connected to the soil with one end in the soil, with the other end of the tube at the same height in open air at normal air pressure (of 1 bar, 1000 hPa or 1000 hN/m^2). A soil with a hydraulic head of -100 cm and a pF of 2 will be able to lower the water level to 100 cm below the top of the tube. Water has a mass of 1 kg/l, hence a column of 100 cm water will apply a pressure of 1000 kg m^2 × 10 N/kg = 10 kN m^{-2} and 100 hPa. This pressure is, of course, the same for both sides of the u-shaped tube. The lower water level on one side means that the soil is compensating for exactly that pressure. This pF value of 2 therefore also equates to a negative pressure of 100 hPa and 0.1 bar. It also require a pressure equivalent to a water column of at least 100 cm to push water out of the pores: the principle used by the pressure membrane apparatus as shown below.



Pressure membrane apparatus operating at -8 to -200 m water column. (after Korzilius (1987))

The relationship between volumetric water content and pF is soil specific, and depends on pore size distribution which is governed by soil texture and bulk density as mentioned above. Bulk density is ignored in LINTUL2 and most other models as this property is dynamic and difficult to quantify. For example, ploughing will increase to proportional volume of large pores, while trampling or heavy machine traffic increase bulk density and decrease the volume of large pores. The pF curve represents, once again, the average soil water retention capacity ignoring these shorter-term dynamics.

Information about specific points on this relationship can inform about the volume of water that a soil can supply to crops for a specific rooting depth; when plants run into oxygen deficiency; and how much water can be extracted with evaporation. These characteristic points on the soil water characteristic are : **air dry** (*AD*) at pF=5; **wilting point** (*WP*) at pF=4.2; **field capacity** (*FC*) at pF=2; and the water content at which the soil **lacks oxygen** (*WET*), at a pF=0.5.

Tipping bucket approach

Two main approaches to model the soil water balance may be distinguished: the **tipping bucket approach** (Keulen, 1975; Keulen and Laar, 1986), and the **Richards approach** based on work from Darcy and Richards, see e.g. van Dam and Feddes (2000). An extensive comparison between the tipping bucket approach and the Richards approach can be found in (Rijneveld, 1996). The Richards approach is based on the Richards-Darcy equation where water flows are computed from pressure head gradients. Transport of water in the soil is driven by vertical, and in more advanced models also horizontal, gradients in matric potentials. Such an approach needs a full pF curve, and a detailed description of the hydraulic conductivity.

The hydraulic conductivity curve relates the change in the speed of water transport (with dimensions length per unit of time, e.g. with a unit of cm d^{-1}) to the matric suction (pF).

The tipping bucket approach only needs information about the soil-specific water contents at these above mentioned characteristic points of the pF curve. The tipping bucket is an approach to keep track of the amount of water in one or more soil compartments, where water is only flowing from the top to lower compartments when it is "tipping over", i.e. when the water content exceeds field capacity (it is sometimes also named the 'cascading model'). If the objective is to calculate the amount of water available to the crop over longer periods of time such as a season, the tipping bucket model is appropriate.

In the tipping bucket approach, the water infiltrating into the soil fills the compartments to field capacity from the surface downwards. A possible surplus of water is lost by deep drainage below the rooting depth. Water entering the profile is distributed instantly, i.e. within one time step of integration of usually one day in crop growth models. Thus, field capacity may be reached within one day. Transport of water between soil compartments along developing gradients in matric potentials is not described in the tipping bucket model. Differences in matrix suction within a soil compartment are also ignored, which seems reasonable as roots will even-out pressure gradients in the soil.

In some models, a tipping bucket approach is used that includes a parameterised method, derived from a detailed physically based Richards model, to "mimick" water transport (Van Keulen, 1975). The water loss by evaporation through the soil surface is withdrawn from the various compartments as a function of soil physical properties and the current water distribution in the soil profile. In the straightforward tipping bucket approach, water stress can be considered, but the consequences of waterlogging and related anoxic conditions on root growth to a lesser extent. However, with respect to waterlogging a number of versions have been developed in which waterlogging and capillary rise are mimicked, while maintaining the comfortable, large daily time step.

Not all the water that reaches the surface infiltrates permanently into the soil, especially not during heavy rain. If more water enters the soil than can be retained at field capacity, the excess is drained below the root zone. Drainage to deeper soil is limited by the maximum drainage rate, a high value implies complete drainage. A low value implies restricted drainage and hence waterlogged conditions may occur during wet periods. A zero value means no drainage at all (impermeable layer). Runoff occurs when the amount of rainfall exceeds the available capacity of the bucket and the excess amount cannot be drained, i.e. exceeds maximum drainage rate.

Soil water balance

A soil water balance is needed to keep track of the amount of water that is in the soil. The soil water balance is determined by infiltration into the soil as a result of precipitation, irrigation, run-off or run-on, and percolation to or capillary rise from deeper soil layers, and actual evaporation and transpiration of water from the soil surface and the crop.

Rain and interception of rain

The amount of **rainfall intercepted by the canopy** (R_{int}, mm d^{-1}) equals the **interception capacity of leaves** (0.25 mm d^{-1}) times the **leaf area index** (LAI). Obviously, this maximum amount can only be intercepted if **rainfall intensity** (mm d^{-1}) is higher. It is assumed that each new day with rain, this interception may take place, or, differently stated, that at the start of a new day the leaves are dry again. The effective rain is therefore the difference between rainfall and the intercepted amount. Interception of rain is described as a function of leaf area index:

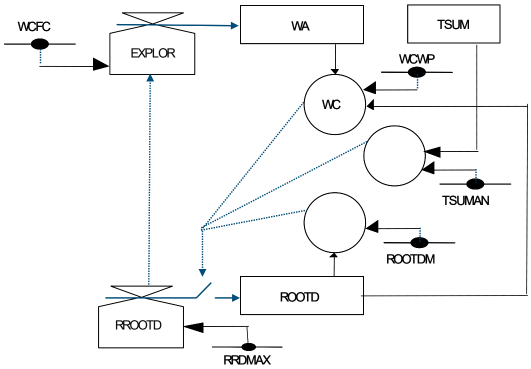
$$R_{int} = min(R, R_{intmax} \times LAI) \tag{16}$$

where R_{intmax} is the **coefficient of maximum interception of rain per day and per unit leaf area**, expressed in mm d^{-1} (liter m^{-2} of leaf d^{-1}), with numerical values reported between 0.03 and 0.6. Equation 16 shows that each day, a **maximum interception** (R_{intmax} times LAI) may take place, which means that it is assumed that at the start of the day leaves are dry.

Drainage, runoff and irrigation

The rate of change (mm d^{-1}) in the amount of water in the bucket needs to be computed. This rate is determined by the **net amount of rainfall**, which is the rainfall minus the intercepted amount that does not reach the soil surface, e.g. water intercepted by leaves, minus the runoff that never gets into the bucket. The amount of water in the bucket also increases with irrigation (**I**, mm d^{-1}) and when roots explore new soil, which also contain some water.

Exploration of water by growing roots, dw_{exp}/dt, depends on the rate of vertical extension of the roots, because new soil water, by definition held at the water content belonging to field capacity, is made available for uptake. Thus dw_{exp}/dt is related to the vertical root extension rate, dr_d/dt, There is either a maximum, crop type-specific and fixed, vertical root growth rate $(dr_d/dt)_{max}$ or no growth at all. Root growth only occurs if there is enough water in the profile, the temperature sum at anthesis, $T_{sum\text{-}ant}$, is not yet reached, and the maximum rooting depth, r_{dmax}, is not yet reached (r_d is the actual rooted depth). Maximum rooting depth is determined by the smallest of either a crop-specific physiological maximum, r_{dmax_crop}, or a soil physical maximum, r_{dmax_soil}, for example when the soil is shallow, because its parent material is rocky. By multiplying dr_d/dt by θ_{fc} the water exploration rate is calculated, which is added to the soil water in terms of mm of water. The relationship between the rate of increase of root biomass, dW_{rt}/dt (g DM m^{-2} d^{-1}), root length and vertical root extension rate, dr_d/dt is complex and not well understood yet. The relational diagram below shows a feedback from the amount of water, WA, to rate $EXPLOR$ (dw_{exp}/dt).



It was stated earlier that the tipping bucket model does not include capillary rise. Usually, this upward flow of water is slow compared to the downward root growth rate before anthesis[1]. After anthesis, capillary rise could provide a small contribution to the water supply of crops, but this is thus neglected in the model.

Note that here, irrigation is the net amount that reaches the soil surface. All components in this rate equation are in mm d^{-1}:

$$\frac{dW}{dt} = Rain + I + exploration - R_{int} - run\,off - \,drainage - T_{act} - E_{act} \tag{17}$$

Drainage of water[1], runoff and irrigation are calculated in a preferential sequence in the tipping bucket model. Irrigation in the LINTUL2 model may be used to easily compare results obtained under rainfed conditions with results obtained under potential production conditions, because irrigation is used to remove water stress. Drainage of water occurs when the balance of incoming and outgoing water of the soil { $R – (R_{int} + T_{act} + E_{act})$ } exceeds the amount of water needed to replenish the soil water from the actual amount, w_t, to the field capacity value, w_{fc}. This is the principal definition of the tipping bucket model. However, the LINTUL2 tipping bucket is made more realistic by a restriction where drainage cannot exceed the maximum drainage rate, D_{max}. Typical maximum drainage rates under saturated conditions are 250-500, 10-35 and 2-3 mm d^{-1}, for sand, clay and heavy clay respectively (Van Keulen and Wolf, 1986)[1]. Runoff of water may occur when the drainage capacity of the soil is not sufficient to discharge the rain water with the result that the soil becomes saturated. Irrigation may be applied in case of water shortage, for example using sprinkler irrigation or overland flow (furrow irrigation). In case of overland flow, there will be no interception of water by the canopy. In the LINTUL2 model, however, irrigation is not modelled as an additional input of water from intermittent applications, e.g. after a certain low soil water content is reached (usually the water content belonging to a pF ≈ 3), but rather as the volume of irrigation water needed to keep the actual water content θ at field capacity, θ_{fc}. In this way, water-limited or rainfed crop production can easily be compared with potential production.

The values for **transpiration** (T_{act}) and **evaporation** (E_{act}) are the actual amounts, in mm d^{-1}, that differ from the potential amounts that were explained above. Below is explained how T_{act} and E_{act} are computed using information about the volumetric water content (which equals the water amount in the bucket divided by its volume). There is mutual dependency here: the T_{act} and E_{act} value depends on water availability for the crop, while the change in water content in the bucket soil depends on T_{act} and E_{act} values. In a numerical model including states and rates, such as LINTUL, dependencies like this can be solved easily in an iterative fashion by calculating the rates of change for each time step (here a day) first before determining the new states. This means that the values of T_{act} and E_{act} on day *i* are determined using the amount of water in the bucket at the start of day *i*. The amount of water on day *i+1* is determined by numerical integration, i.e. $W_{i+1} = W_i + \Delta t \times \frac{dW}{dt}$, where Δt reflects the numerical time step.

LINTUL2 A SIMPLE GENERAL CROP GROWTH MODEL FOR WATER-LIMITED GROWING CONDITIONS (EXAMPLE: SPRING WHEAT)

LINTUL2 builds on LINTUL1 and includes a tipping bucket model and uses the Penman equation to estimate potential evapotranspiration. The simple tipping bucket water balance in LINTUL2 is derived from more complex versions documented by Stroosnijder (1982) and (Penning de Vries, 1989). In LINTUL2, it contains one soil layer, and all of the above mentioned flows, except capillary rise. The single rooted soil layer increases in thickness with the rate at which these roots grow in downward direction and thus soil, in which the water content equals that belonging to field capacity, is explored for the soil water newly found. Figure 4 summarizes the flows and shows which ones are affected by *LAI*.

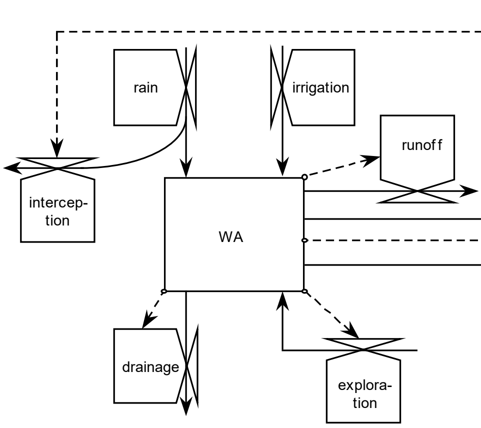


Figure 4. Relational diagram of the tipping bucket water balance in LINTUL2, comprising the amount of water (WA) in the single rooted soil layer as state variable and the water flows into and out of this soil layer. The feedback of the crop on the appropriate flows is indicated by dashed lines starting at LAI.

The length of fibrous roots can vary strongly without much dependence on root weight. Hence, rooted depth is calculated independently of the growth of root mass. This also means that extra root growth, resulting in a large weight of roots, has no effect on rooting depth or water uptake in the LINTUL2 model. The rate at which the rooted depth increases varies between 10 and 30 mm d^{-1} depending on soil and crop characteristics. For spring wheat a value of 12 mm d^{-1} is common (Keulen and Seligman, 1987). Root growth generally stops around flowering in spring wheat (TSUM = TSUMAN), but earlier if the soil becomes too dry ($\theta < \theta_{WP}$) or if the simulated cultivar has reached its maximum depth for the particular soil type.

Actual transpiration

The **actual transpiration** (T_{act}) is computed as:

$$T_{act} = T_{red-tran} \times T_{crop} \quad (18)$$

Where the $T_{red-tran}$ is the transpiration reduction factor with values between 0 (no growth due to drought) and 1 (unlimited growth). This $T_{red-tran}$, and crop transpiration, has a value of one and is declining linearly when the soil water content is below a critical water content (θ_{cr}) in case of water shortage, and above a critical water content (θ_{wet}, compare the water content at "*WET*" in Figure 3) in case of a surplus of water, see Figure 5. The latter case reflects a shortage of oxygen that occurs in very wet soils for most plant species, except for aerenchymatic species such as rice and reed.

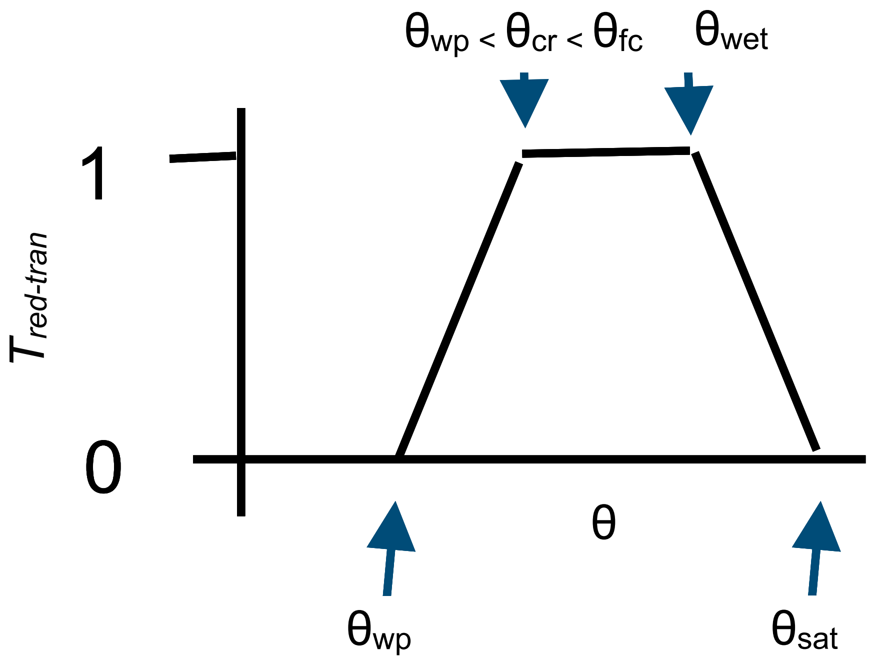


Figure 5. The transpiration reduction factor ($T_{red\text{-}tran}$, y-axis) as a function of water content (θ, x-axis) between wilting point (θ_{wp}) and completely saturated soil (θ_{sat}). Below θ_{cr} and beyond θ_{wet}, transpiration reduction takes place.

The **critical water content** (θ_{cr}) lies between the **water content at field capacity** (θ_{fc}) and that at **wilting point** (θ_{wp}). It is assumed that θ_{cr} is affected by the drought tolerance of the crop: a very drought-tolerant crop will unrestrictedly extract water from the soil to a lower water content as compared to a drought-sensitive crop.

The **critical water content** is calculated by Eq. (17):

$$\theta_{cr} = \theta_{wp} + \frac{T_{crop}}{T_{crop} + TRANCO} \times (\theta_{fc} - \theta_{wp}) \qquad (19)$$

where T_{crop} is the **potential transpiration rate of the crop** as computed with the Penman equation, and *TRANCO* is the **transpiration coefficient**, which is a measure of the drought tolerance of the crop. From equation 19 it is clear that the critical water content will be higher when the value for TRANCO is lower for the same value of T_{crop}. This means that a crop with a lower TRANCO will more quickly respond to drought conditions, i.e. is more sensitive to drought than a crop with a higher TRANCO.

The ratio T_{crop} / (T_{crop} + *TRANCO)* in equation 19 is a fraction between 0 (at an infinite drought tolerance, where $\theta_{cr} = \theta_{wp}$) and 1 (if the crop would be extremely sensitive to drought, resulting in $\theta_{cr} = \theta_{fc}$). By including the dynamic potential transpiration rate in the calculation of θ_{cr} we recognize that on days with a high potential transpiration rate it is more difficult for even a drought-tolerant crop to take up enough water from the soil and to maintain its turgor against the large demand compared to days with a low potential transpiration rate.

Actual evaporation

Transpiration completely ceases at wilting point, θ_{wp}. Actual evaporation (E_{act}) however, can continue untill the air-dry water content in the soil, θ_{ad}, is reached according to Figure 6.

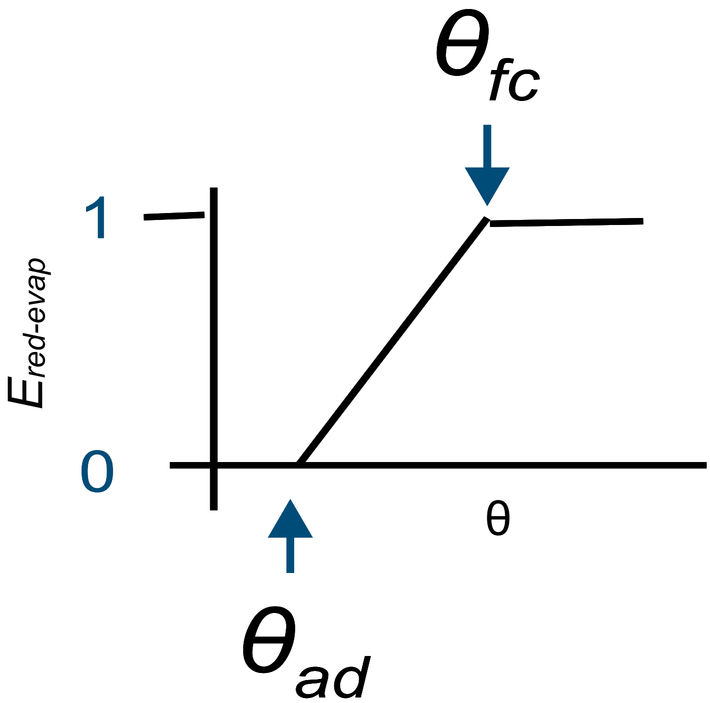


Figure 6. The evaporation reduction factor ($E_{red\text{-}evap}$, y-axis) as a function of water content (θ, x-axis) between the water contents at field capacity (θ_{fc}) and at air dry (θ_{ad}). Below θ_{fc} evaporation reduction takes place.

The evaporation reduction factor, $T_{red\text{-}evap}$ is defined as E_{act} / E_{soil}, and hence E_{act} can be computed as:

$$E_{act} = E_{red-evap} \times E_{soil}. \tag{20}$$

Interactions between crop and soil water

Crop roots react to water tension and water flow rates, rather than to water content. Transport due to gradients in soil water tension is not included in the simple tipping bucket approach. However, this aspect of water availability in relation to crop demand and soil water tension is captured by the transpiration reduction factor ($T_{red\text{-}tran}$).

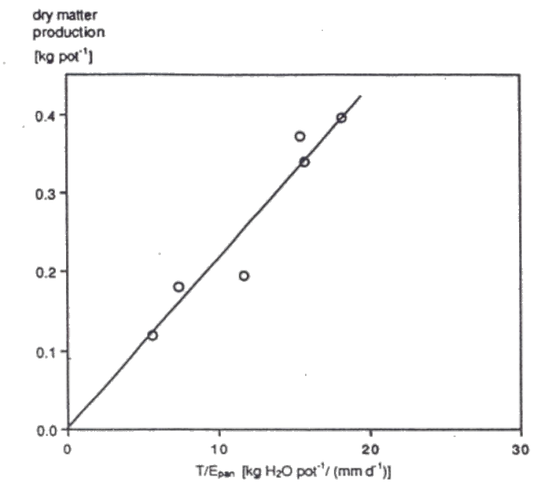
Drought and hence soil water content is expected to affect emergence, root growth, crop growth rate, and the allocation of biomass over roots and shoot of the crop. The transpiration reduction factor ($T_{red\text{-}tran}$), is also used to reduce the leaf expansion rate in the juvenile stage, which is mainly driven by temperature (see LINTUL1 text):

$$\frac{dL}{dt} = \frac{L_t \times (e^{(r_l \times T_{eff} \times \Delta t)} - 1)}{\Delta t} \times T_{red-tran} \tag{21}$$

Moreover, crop growth rate as a whole is reduced. From experiments of Briggs & Shantz (1914) with maize it appears that such a reduction in growth is proportional to the ratio of the actual transpiration and the potential transpiration (Figure 6). From equation 16 it is clear that T_{red_tran} is equal to this ratio and hence can be used to reduce crop growth rates:

$$\frac{dW}{dt} = RUE \times I_{int} \times T_{red-tran} \quad (22)$$

Both equations 21 and 22 are extensions of the equations 12 and 3 as provided in the description ofLINTUL1, respectively. The use of the transpiration reduction factor as a multiplication factor in equation 22 reflects the combined effects of closure of stomata on CO_2 exchange, leaf rolling and leaf angle changes on light interception, and reduced radiation use efficiency under drought conditions that may be due to e.g. reduced conversion of assimilates to dry matter or assimilate transport in the plant.



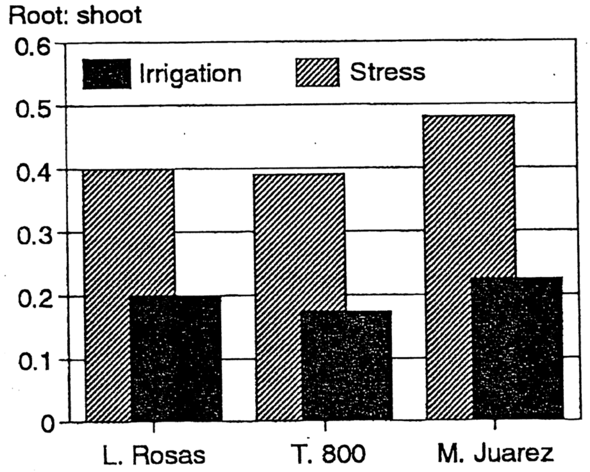
[7], Briggs & Shantz (1914), for maize. The ratio on the x-axis may be called the transpiration reduction factor, $T_{red-tran}$. The $T_{red-tran}$ is here given in kg H_2O pot^{-1} / (mm d^{-1}), but in the further text the units are expressed in (mm d^{-1}) / (mm d^{-1}).

Furthermore, emergence and root growth take place only if there is enough water in the soil, i.e. if the water content is above wilting point. Emergence takes place only once, of course, but in the course of crop development, root growth could be hampered a number of times due to a water content below wilting point.

Allocation of biomass over roots and shoot of the crop

Allocation of biomass over roots and shoot of the crop is changed if water stress occurs, Figure 7 (Magrin *et al.*, 1991).

[7] E_{pan} is the evaporation from an open pan filled with water and is a measure for potential evapotranspiration.



The crop thus responds to a water shortage by a better exploration of the soil volume. Note that in the LINTUL2 model, more root biomass does not affect soil water uptake. Since allocation of biomass over the different crop parts is not well understood, the process is modelled in a general way, based on two assumptions: (1) upon drought, more roots will be formed to alleviate the water shortage and thus less biomass is left for the shoot, (2) the distribution of dry matter among stem, leaves and storage organs remains unchanged.

Biomass is either allocated to roots or to shoots (including leaves, stems and storage organs for spring wheat) and:

$$sh_{old} = 1 - r_{old} \quad (23)$$

When the soil is too dry, more roots are formed to alleviate the water shortage and thus more newly produced biomass is allocated to the roots. The new amount of biomass allocated to the roots is then:

$$r_{new} = r_{old} \times rMOD, \text{ with } 1 \leq rMOD \leq f_{max} \quad (24)$$

where *rMOD* is the root modification factor and f_{max} is the maximum modification of the partitioning of dry matter to the roots. If more of the assimilates are allocated to roots, less will be available for the shoot:

$$sh_{new} = 1 - r_{new} \quad (25)$$

The **new shoot allocation factor** can also be expressed as **a fraction of the old shoot allocation factor**:

$$sh_{new} = shMOD \times sh_{old} \quad (26)$$

Rewriting the equations above, results in:

$$sh_{old} = 1 - \frac{r_{new}}{rMOD} \quad (27)$$

And when combining the above equations, the shMOD factor can be derived:

$$shMOD = \frac{sh_{new}}{sh_{old}} = \frac{1-r_{new}}{1-r_{new}/rMOD} \quad (28)$$

What remains to be defined is an equation for **the modification factor** *rMOD*. Equation 24 states: 1 $\leq$ *rMOD* $\leq$ f_{max}, where the number 1 represents no modification in allocation of dry matter over root and shoot and the variable f_{max} represents the maximum modification in allocation of dry matter to the roots. A numerical value of f_{max} of 2 seems reasonable in view of Figure 7, although we do not know how severe the water stress in the experiments of (Magrin ***et al.***, 1991) was. Therefore, f_{max} could be set to a different value, if appropriate. The function for *rMOD* is defined by a hyperbola type of equation:

$$rMOD = \begin{cases} \frac{1}{T_{red-tran}+\frac{1}{f_{max}}}, & \text{if } T_{red-tran} \leq \frac{1}{f_{max}} \\ 1, & \text{if } T_{red-tran} > \frac{1}{f_{max}} \end{cases} \quad (29)$$

Equation 29 logically comprises the restriction that *rMOD* cannot be smaller than 1. Figure 8 shows plots of equation 29 for f_{max} = 2 and f_{max} = 4. The function intersects the y-axis at a value f_{max}, and the x-axis at a value (1-1/f_{max}), beyond which $T_{red\text{-}tran}$ does not affect the partitioning fractions. The higher f_{max}, the earlier the crop experiences water stress, and the more dry matter is distributed to the roots. Enhanced allocation of dry matter to roots only occurs under drought stress, meaning that equation 29 holds when $\theta < \theta_{cr}$. In case of waterlogging, inducing oxygen stress, literature suggests that dry matter allocation to roots is *likely* to be reduced (Davis ***et al.***, 1994). This phenomenon needs more study and is therefore not included in the LINTUL model.

Let r_{old}, s_{old}, l_{old}, and so_{old}, denote the partitioning fractions for roots, stem, leaves and storage organs under non water-limited conditions, and r_{new}, s_{new}, l_{new}, and so_{new}, the partitioning fractions under water-limited conditions. Then, the partitioning fractions under water limited conditions can be calculated by:

$$r_{new} = r_{old} \times rMOD \quad (30)$$

$$s_{new} = s_{old} \times shMOD \quad (31)$$

$$l_{new} = l_{old} \times shMOD \quad (32)$$

$$so_{new} = so_{old} \times shMOD \quad (33)$$

where *rMOD* and *shMOD* are given by equations 28 and 29.

In LINTUL, the growth of crop organs is computed as:

$$\frac{dW_i}{dt} = RUE \times I_{int} \times F_i \quad (34)$$

where the partitioning fractions, F_i, were defined for conditions without drought stress and *i* represents the root (*rt*), stem (*st*), leaves (*lv*) or storage organ (*so*) of the plant, respectively. Hence F_i reflects r_{old}, s_{old}, l_{old}, and so_{old}. However, the symbol F_i here refers to the modified fractions r_{new}, s_{new}, l_{new}, and so_{new}. Obviously, the *new* and the *old* partitioning fractions coincide if *rMOD* = 1 (no water shortage and thus no modification), resulting in *shMOD* = 1.

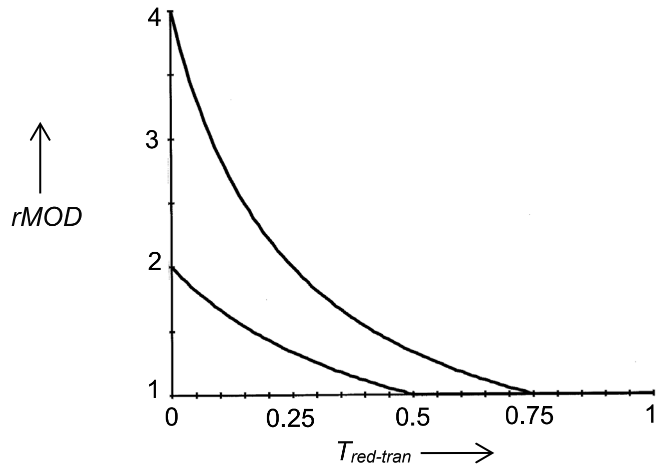


Figure 8. The root modification factor rMOD as a function of $T_{red\text{-}tran}$ for f_{max} = 2 (lower line) and f_{max} = 4 (upper line). Note that the origin is (0,1) rather than (0,0).

Figure 9 summarizes the interactions between crop and soil water, where, feedbacks of *LAI* on the water balance and the feedbacks of the transpiration reduction factor, $T_{red\text{-}tran}$ (TRANRF), on *dL/dt*, *GLAI* and *dW/dt* (*GTOTAL*) are indicated (for the definitions of the abbreviations see the table 3 at the end of this chapter).

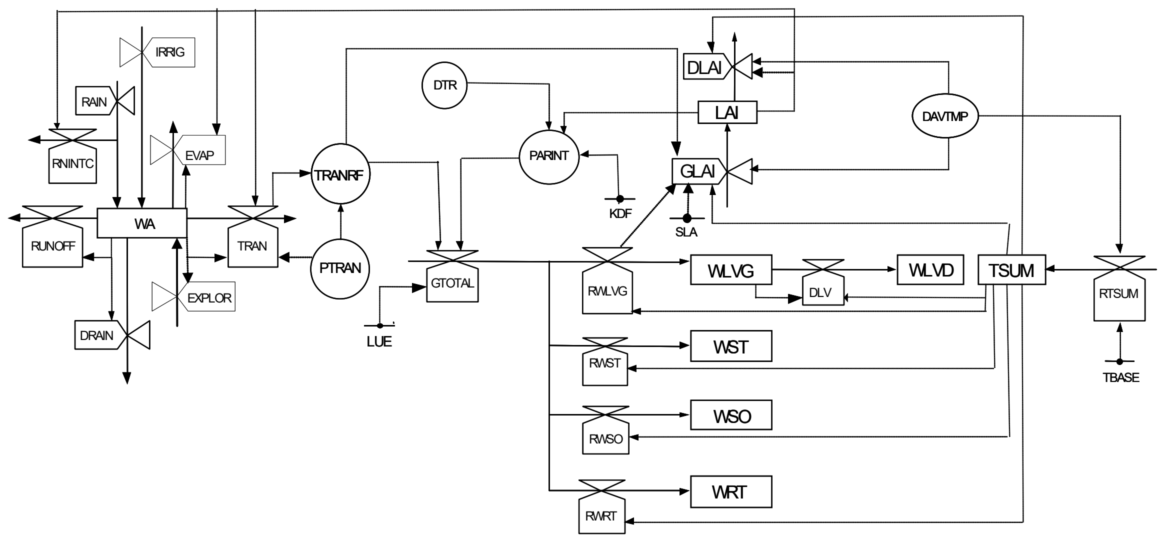


Figure 9. Relational diagram combining the potential crop growth model with the water balance model to obtain LINTUL2. Note especially the feedbacks of the water balance, via the transpiration reduction factor, TRANRF ($T_{red\text{-}tran}$), on GLAI (dL/dt) and on GTOTAL (dW/dt). Moreover, potential transpiration (PTRAN, T_{crop}), needed to calculate the dimensionless TRANRF ($T_{red\text{-}tran}$), is indicated.

The abbreviations in Table 3 reflect the names as used in the implementation of the model in code, as also shown in figure 9. The default parameters for spring wheat are given in Table 4.

Table 3. Some important corresponding abbreviations in the mathematical description.

Mathematical abbreviation	*Description*	*Unit*	*Program abbreviation*
ET	Potential evapotranspiration of the crop–soil system as a whole	kg H_2O m^{-2} d^{-1} or mm H_2O d^{-1}	No separate variable
T_{crop}	Potential transpiration of the crop	kg H_2O m^{-2} d^{-1}	*PTRAN*
E_{soil}	Potential evaporation of the soil	kg H_2O m^{-2} d^{-1}	*PEVAP*
T_{act}	Actual transpiration of the crop	kg H_2O m^{-2} d^{-1}	*TRAN*
E_{act}	Actual evaporation of the soil	kg H_2O m^{-2} d^{-1}	*EVAP*
$T_{red\text{-}tran}$	Transpiration reduction factor	–	*TRANRF* Also FR in subroutine EVAPTR.
$E_{red\text{-}evap}$	Evaporation reduction factor	–	Calculated variable: $(\theta - \theta_{ad}) / (\theta_{fc} - \theta_{ad})$, where $0 <= T_{red\text{-}evap} <=1$

Table 4. Names and descriptions of parameters in LINTUL2 with units and default values for Spring Wheat.

Parameter	*Description*	*Unit*	*Value*
		LINTUL1	
RUE	Radiation use efficiency	gDM MJ^{-1}(PAR)	3
fPAR	Fraction of photosynthetic active radiation	MJ(PAR) MJ^{-1}(TDR)	0.5
k	Light extinction coefficient	-	0.6
SLA	Specific leaf area	m^2 g^{-1}	0.022
RRDMAX	Maximum relative root depth rate	m d^{-1}	0.012
TBASE	Base temperature	°C	0
RGRL	Relative growth rate of leaves	$(°Cd)^{-1}$	0.009
TSUMAN	Temperature sum at anthesis	°Cd	1110
FINTSUM	Temperate sum at full maturity	°Cd	2080
LAICR	Critical leaf area index	m^2(leaf) m^2(soil)	4
RDRSHM	Relative death rate at maximum shading	d^{-1}	0.03
		LINTUL2	
MMWET	Maximum amount of intercepted rainfall	mm	0.25
TRANCO	Transpiration coefficient	mm d^{-1}	8
ROOTDI	Initial rooting depth	m	Depends on soil
ROOTDM	Maximum rooting depth	m	Depends on soil
WCI	Initial water content	$m^3(H_2O)$ m^{-3} (soil)	Depends on soil
WCAD	Water content at air dry	$m^3(H_2O)$ m^{-3} (soil)	Depends on soil
WCWP	Water content at wilting point	$m^3(H_2O)$ m^{-3} (soil)	Depends on soil
WCFC	Water content at field capacity	$m^3(H_2O)$ m^{-3} (soil)	Depends on soil
WCWET	Water content when wet	$m^3(H_2O)$ m^{-3} (soil)	Depends on soil
WCST	Water content at saturation	$m^3(H_2O)$ m^{-3} (soil)	Depends on soil
DRATE	Maximum drainage rate	mm d^{-1}	Depends on soil

References

Allen, R.G., Pereira, L.S., Raes, D., Smith, M., 1998. Crop evapotranspiration-Guidelines for computing crop water requirements-FAO Irrigation and drainage paper 56. FAO, Rome 300, D05109.

Davis, T.D., Haissig, B.E., International Symposium on the Biology of Adventitious Root, F., 1994. Biology of adventitious root formation. Plenum Press, New York.

Howell, T.E., Evett, S.R., 2004. The Penman-Monteith Method. . Section 3 in Evapotranspiration: Determination of Consumptive Use in Water Rights Proceedings. Continuing Legal Education in Colorado, Inc. . USDA, Denver, CO.

Keulen, H.v., 1975. Simulation of water use and herbage growth in arid regions. Pudoc, Wageningen.

Keulen, H.v., Laar, H.H.v., 1986. Modelling of agricultural production : weather, soils and crops. Pudoc, 1986 (Simulation monographs), Wageningen.

Keulen, H.v., Seligman, N.G., 1987. Simulation of water use, nitrogen nutrition and growth of a spring wheat crop. Pudoc, Wageningen.

Koorevaar, P., menelik, G., Dirksen, C., 1983. Elements of soil physics. Elsevier Science Amsterdam, the Netherlands.

Korzilius, E., 1987. Influence of hysteresis in the soil water characteristics on water redistribution in a cylindrical soil aggregate. Korzilius, [S.l.].

Kraalingen, D.W.G.v., Stol, W., 1997. Evapotranspiration modules for crop growth simulation : implementation of the algorithms from Penman, Makkink and Priestley-Taylor. DLO Research Institute for Agrobiology and Soil Fertility ;

The C.T. de Wit Graduate School for Production Ecology, Wageningen.

Magrin, G.O., Hall, A.J., Castellano, S., Meira, S.G., 1991. Rooting depth, growth cycle duration, and timing of the jointing stage in wheat: Traits that can contribute to early drought tolerance. In: Saunders, D.A. (Ed.), Wheat for the nontraditional warm areas : a proceedings of the international conference, July 29-August 3, 1990, Foz do Iguaçu, Brazil. International Maize and Wheat Improvement Center, Lisboa.

Penman, H.L., 1948. Natural Evaporation from Open Water, Bare Soil and Grass. Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences 193, 120-145.

Penman, H.L., 1956. Evaporation: an introductory survey. NJAS-Wagen. J. Life Sc. 4, 9-29.

Penning de Vries, F.W.T., 1989. Simulation of ecophysiological processes of growth in several annual crops. Pudoc, Wageningen :.

Rijneveld, W., 1996. On water flow : simulation of unsaturated water flow with the tipping bucket model with special regard to the simulation of hysteresis and preferential flow. Rijneveld, [S.l.].

Singh, B., Szeicz, G., 1979. The effect of intercepted rainfall on the water balance of a hardwood forest. Water Resour. Res. 15, 131-138.

Stroosnijder, L., 1982. Simulation of the soil water balance. Simulation of plant growth and crop production; ISBN: 9789022008096. Pudoc 1982.

van Dam, J.C., Feddes, R.A., 2000. Numerical simulation of infiltration, evaporation and shallow groundwater levels with the Richards equation. Journal of Hydrology 233, 72-85.

Van Keulen, H., Wolf, J., 1986. Modelling of agricultural production: weather, soils and crops. Centre for Agricultural Publishing and Documentation., Wageningen, the Netherlands.